

FURTHER STUDY ON THE PROPERTIES OF OPERATORS OF ATMOSPHERIC EQUATIONS AND THE EXISTENCE OF ATTRACTOR*

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Received April 18, 1996; revised September 11, 1996

ABSTRACT

The equivalent operator equation is derived from the full primitive nonlinear equations of the atmospheric motion and the properties and physical senses of the operators are studied. In the infinite dimensional Hilbert space, the global asymptotic behavior of the atmosphere system with the non-stationary external forcing is studied under the assumption of the bounded external forcing. The existence theorems of the global absorbing set and the global attractor are obtained. Thus, the conclusions deduced from the large-scale atmosphere (Li and Chou 1996 a; 1996 b) are extended to the general atmosphere.

Key words: operator equation, global attractor, non-stationary forcing

I. INTRODUCTION

The atmosphere is a forced dissipative open system. The most basic physical characteristics of its long-range process are the diabatic and the dissipative, namely, the energy supplement and the energy dissipation. The atmospheric motion can be maintained because of the continuous energy exchange between the exterior of the atmosphere and the interior of the atmosphere. Therefore, it is not suitable to discuss the long-range weather process by using the equations with adiabatic approximation and omitting friction.

Based on the equations of large-scale atmospheric motion with the external forcing and dissipation, Chou (1983; 1986; 1990) deduced the corresponding operator equation and discussed the properties of operators. He concluded that the atmosphere system has the characteristic in the decay of the effect of initial field. Afterwards, under the stationary forcing (Wang et al. 1989; Lions et al. 1992) and the non-stationary forcing (Li and Chou 1996 a; 1996 b), the existence of the attractor for the equations of large-scale atmosphere was proved and some significant results were obtained. Additionally, Lions et al. (1992) also estimated the Housdoff dimension of the attractor.

To sum up, all of the above studies have been carried out based on the large-scale atmospheric motion and adopted quasi-hydrostatic approximation. Apparently, the atmosphere is a multi-scale motion system that does not confine to large-scale motion.

* The project is supported by the State Key Research Project on Dynamics and Predictive Theory of the Climate.

Although the quasi-hydrostatic approximation is satisfied with high accuracy in the vertical of the atmosphere, it is after all a simplification of equation of vertical motion. Strictly speaking, the primitive form of the equation of atmospheric motion in the vertical is not the equation of static equilibrium. Then, whether or not the conclusions mentioned above are true for the full primitive equations without the simplifications of large-scale and quasi-hydrostatic approximation is still a problem. Evidently, it is worthy of further work. This paper will present a theoretical attempt of making an approach to it.

II. BASIC EQUATIONS

In the spherical coordinate system $(\lambda, \theta, r; t)$ (λ is the longitude, θ the colatitude, r the geocentric distance), the full primitive equations of the atmosphere can be written as

$$\left\{ \begin{array}{l} \frac{\partial V_\lambda}{\partial t} + \Lambda V_\lambda + (f + \frac{V_\lambda}{r} \operatorname{ctg} \theta) V_\theta + (\mathcal{F} + \frac{V_\lambda}{r}) V_r + \frac{1}{r \sin \theta} \frac{R}{\rho} \frac{\partial \rho T}{\partial \lambda} - F_\lambda = 0, \\ \frac{\partial V_\theta}{\partial t} + \Lambda V_\theta - (f + \frac{V_\lambda}{r} \operatorname{ctg} \theta) V_\lambda + \frac{V_\theta}{r} V_r + \frac{1}{r} \frac{R}{\rho} \frac{\partial \rho T}{\partial \theta} - F_\theta = 0, \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} \frac{\partial V_r}{\partial t} + \Lambda V_r - (\mathcal{F} + \frac{V_\lambda}{r}) V_\lambda - \frac{V_\theta}{r} V_\theta + g + \frac{R}{\rho} \frac{\partial \rho T}{\partial r} - F_r = 0, \\ \frac{\partial \rho}{\partial t} + \Lambda \rho + \rho \left(\frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) = 0, \end{array} \right. \quad (2)$$

$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \Lambda \rho + \rho \left(\frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) = 0, \\ C_v \left(\frac{\partial T}{\partial t} + \Lambda T \right) + RT \left(\frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) = \frac{\epsilon}{\rho}, \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} C_v \left(\frac{\partial T}{\partial t} + \Lambda T \right) + RT \left(\frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) = \frac{\epsilon}{\rho}, \end{array} \right. \quad (4)$$

$$\left\{ \begin{array}{l} C_v \left(\frac{\partial T}{\partial t} + \Lambda T \right) + RT \left(\frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) = \frac{\epsilon}{\rho}, \end{array} \right. \quad (5)$$

where

$$f = 2\Omega \cos \theta,$$

$$\mathcal{F} = 2\Omega \sin \theta,$$

$$\Lambda = \mathbf{V} \cdot \nabla = \frac{V_\lambda}{r \sin \theta} \frac{\partial}{\partial \lambda} + \frac{V_\theta}{r} \frac{\partial}{\partial \theta} + V_r \frac{\partial}{\partial r},$$

$$F_\lambda = \frac{\nu}{3} \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \lambda} \frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{\partial}{\partial \lambda} \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{\partial}{\partial \lambda} \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) + \nu \Delta V_\lambda,$$

$$F_\theta = \frac{\nu}{3} \frac{1}{r} \left(\frac{\partial}{\partial \theta} \frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{\partial}{\partial \theta} \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{\partial}{\partial \theta} \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) + \nu \Delta V_\theta,$$

$$F_r = \frac{\nu}{3} \left(\frac{\partial}{\partial r} \frac{1}{r \sin \theta} \frac{\partial V_\lambda}{\partial \lambda} + \frac{\partial}{\partial r} \frac{1}{r \sin \theta} \frac{\partial V_\theta \sin \theta}{\partial \theta} + \frac{\partial}{\partial r} \frac{1}{r^2} \frac{\partial r^2 V_r}{\partial r} \right) + \nu \Delta V_r,$$

$$\Delta = \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \lambda^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r},$$

$$\nu = \mu/\rho,$$

μ is the molecular viscosity coefficient, ϵ the diabatic heating, all other notations are as usual meteorologically.

The domain of solutions of Eqs. (1) – (5) is $\Omega = S^2 \times (r_s, r_\infty)$ with $0 < r_s < r_\infty < \infty$. Here $r_s = r_s(\lambda, \theta)$ is the distance between the surface of the earth at the longitude of λ and the colatitude of θ and the geocentric, r_∞ is a certain large number. The boundary value conditions are given below.

On the earth's surface $r = r_s$,

$$(V_\lambda, V_\theta, V_r) = 0, \quad (6)$$

$$\frac{\partial T}{\partial r} = \alpha_s(T - T_s). \quad (7)$$

If we consider orographic effect, Eq. (6) should be rewritten as

$$\begin{cases} \frac{\partial}{\partial r}(V_\lambda, V_\theta) = 0, \\ V_r = \mathbf{V}_{hs} \cdot \nabla_h r_s = \frac{V_\lambda}{r \sin \theta} \frac{\partial r_s}{\partial \lambda} + \frac{V_\theta}{r_s} \frac{\partial r_s}{\partial \theta}. \end{cases} \quad (6')$$

In this paper, the conclusions got by Eq. (6) or Eq. (6') are the same. $T_s = T_s(\lambda, \theta; t)$ in Eq. (7) is the temperature on the earth's surface, α_s is a positive constant related to turbulent thermal conductivity, $\alpha_s \in L^\infty(S^2) \cap R_+$.

On the upper surface of the atmosphere $r = r_\infty$,

$$\begin{cases} \rho(V_\lambda^2, V_\theta^2, V_r^2, \Phi, T) = 0, \\ \frac{\partial}{\partial r}(V_\lambda, V_\theta) = 0, V_r = 0, \\ \frac{\partial T}{\partial r} = 0, \end{cases} \quad (8)$$

where $\Phi = gr$.

The initial values are

$$(V_\lambda, V_\theta, V_r, \rho, T)|_{t=0} = (V_\lambda^{(0)}, V_\theta^{(0)}, V_r^{(0)}, \rho^{(0)}, T^{(0)}). \quad (9)$$

III. OPERATOR EQUATION

Introducing the following vector function

$$\varphi = (\tilde{V}_\lambda, \tilde{V}_\theta, \tilde{V}_r, \tilde{\rho}, \tilde{T})', \quad (10)$$

where the sign “'” denotes transposition. $(\tilde{V}_\lambda, \tilde{V}_\theta, \tilde{V}_r, \tilde{\rho}, \tilde{T})' = \rho^* (V_\lambda^*, V_\theta^*, V_r^*, \sqrt{\Phi}, T^*)'$,

$V_\lambda^* = V_\lambda / \sqrt{2}$, $V_\theta^* = V_\theta / \sqrt{2}$, $V_r^* = V_r / \sqrt{2}$, $\rho^* = \sqrt{\rho}$, $T^* = \sqrt{C_v T}$. then Eqs. (1) – (5) (field of turbulent mean flow) can be written as the following equivalent operator equation:

$$\frac{\partial \varphi}{\partial t} + (N(\varphi) + L(\varphi))\varphi = \xi(\varphi), \quad (11)$$

where

$$N(\varphi) =$$

$$\begin{vmatrix} \mathcal{L} & 2\Omega \cos \theta + (V_\lambda/r) \operatorname{ctg} \theta & 2\Omega \sin \theta + V_\theta/r & 0 & l'_1 G \\ -2\Omega \cos \theta - (V_\lambda/r) \operatorname{ctg} \theta & \mathcal{L} & V_\theta/r & 0 & l'_2 G \\ -2\Omega \sin \theta - V_\theta/r & -V_\theta/r & \mathcal{L} & g/\sqrt{2\Phi} & l'_3 G \\ 0 & 0 & -g/\sqrt{2\Phi} & \mathcal{L} & 0 \\ Gl_1 & Gl_2 & Gl_3 & 0 & \mathcal{L} \end{vmatrix},$$

$$L(\varphi) =$$

$$\begin{vmatrix} -(\mu/3)l'_1l_1 - \mu l_4 - l & -(\mu/3)l'_1l_2 & -(\mu/3)l'_1l_3 & 0 & 0 \\ -(\mu/3)l'_2l_1 & -(\mu/3)l'_2l_2 - \mu l_4 - l & -(\mu/3)l'_2l_3 & 0 & 0 \\ -(\mu/3)l'_3l_1 & -(\mu/3)l'_3l_2 & -(\mu/3)l'_3l_3 - \mu l_4 - l & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -l_5 \end{vmatrix},$$

$$\xi(\varphi) = (0, 0, 0, 0, (\varepsilon + C_k \alpha_s T_s)/2\tilde{T})',$$

$$\mathcal{L} = (\Pi + \Lambda)/2,$$

$$\Pi = \frac{1}{r \sin \theta} \frac{\partial}{\partial \lambda} V_\lambda + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} V_\theta \sin \theta + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 V_r,$$

$$G = R\tilde{T}/\sqrt{2}C_v,$$

$$l = \frac{1}{\rho^*} \left(\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \lambda} k_\lambda \frac{\partial}{\partial \lambda} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} k_\theta \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial}{\partial r} k_r r \frac{\partial}{\partial r} \right) \frac{1}{\rho^*},$$

$$l_1 = \frac{1}{r \sin \theta} \frac{\partial}{\partial \lambda} \frac{1}{\rho^*},$$

$$l_2 = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \frac{\sin \theta}{\rho^*},$$

$$l_3 = \frac{1}{r^2} \frac{\partial}{\partial r} \frac{r^2}{\rho^*},$$

$$l_4 = \frac{1}{\rho^*} \Delta \frac{1}{\rho^*},$$

$$l_5 = \frac{1}{\rho^*} \left(\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \lambda} K_\lambda \frac{\partial}{\partial \lambda} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} K_\theta \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial}{\partial r} K_r r \frac{\partial}{\partial r} \right) \frac{1}{\rho^*} - \frac{C_k \alpha_s T_s}{\tilde{T}^2},$$

$$l'_1 = (1/\rho^*) l_1 \rho^*,$$

$$l'_2 = (\sin \theta / \rho^*) l_2 (\rho^* / \sin \theta),$$

$$l'_3 = (r^2 / \rho^*) l_3 (r^2 / \rho^*),$$

$$C_K = K_r r_s^2 / C_r,$$

$$C_r = \int_{r_s}^{r_\infty} r^2 dr,$$

where $k_i (i = \lambda, \theta, r)$ the turbulent viscosity coefficient, and $K_i (i = \lambda, \theta, r)$ the turbulent thermal conductivity.

The initial value may be written as

$$\varphi|_{t=0} = \varphi_0. \quad (12)$$

The boundary value conditions are the same as mentioned before.

IV. PROPERTIES AND SENSES OF THE OPERATORS

Let $H_0(\Omega)$ be the complete space with the inner product and the norm as follows:

$$(\varphi_1, \varphi_2) = \int_{\Omega} \varphi_1' \varphi_2 d\Omega = \int_{r_s}^{r_\infty} \int_0^\pi \int_0^{2\pi} \varphi_1' \varphi_2 r^2 \sin \theta d\lambda d\theta dr, \quad (13)$$

$$\|\varphi\|_0^2 = (\varphi, \varphi)^{1/2}, \quad (14)$$

$\forall \varphi = (\tilde{V}_{i\lambda}, \tilde{V}_{i\theta}, \tilde{V}_{ir}, \tilde{\rho}_i, \tilde{T}_i)', i=1,2$. $H_0(\Omega)$ is a Hilbert space. Let $N^*(\varphi)$ and $L^*(\varphi)$ be the adjoint operators of $N(\varphi)$ and $L(\varphi)$ respectively.

Property 1.

$$N(\varphi) = -N^*(\varphi), \quad (15)$$

$$L(\varphi) = L^*(\varphi). \quad (16)$$

We call $L(\varphi)$ the positive definite self-adjoint operator, and $N(\varphi)$ the anti-adjoint operator.

Property 2. $L(\varphi)$ is symmetric. $N(\varphi)$ is anti-symmetric, i.e.,

$$(\varphi_1, L(\varphi)\varphi_2) = (L(\varphi)\varphi_1, \varphi_2), \quad (17)$$

$$(\varphi_1, N(\varphi)\varphi_2) = - (N(\varphi)\varphi_1, \varphi_2), \quad (18)$$

$\forall \varphi, \varphi_1, \varphi_2 \in H_0(\Omega)$.

Property 3.

$$(\varphi, L(\varphi)\varphi) \geq 0, \quad (19)$$

$$(\varphi, N(\varphi)\varphi) = 0, \quad (20)$$

$\forall \varphi, \varphi_1 \in H_0(\Omega)$. The equality in Eq. (19) is true if and only if $\|\varphi\|_0 = 0$.

Property 4.

$$(\varphi, L(\varphi)\varphi) = (\psi, \tilde{L}\psi), \quad (21)$$

where

$$\psi = (V_{\lambda}^*, V_{\theta}^*, V_r^*, \rho^*, T^*)',$$

$$\tilde{L} = \begin{vmatrix} -\frac{\mu}{3r\sin\theta} \frac{\partial}{\partial \lambda} \tilde{l}_1 - \mu\Delta - \tilde{l} & -\frac{\mu}{3r\sin\theta} \frac{\partial}{\partial \lambda} \tilde{l}_2 & -\frac{\mu}{3r\sin\theta} \frac{\partial}{\partial \lambda} \tilde{l}_3 & 0 & 0 \\ -\frac{\mu}{3r} \frac{\partial}{\partial \theta} \tilde{l}_1 & -\frac{\mu}{3r} \frac{\partial}{\partial \theta} \tilde{l}_2 - \mu\Delta - \tilde{l} & -\frac{\mu}{3r} \frac{\partial}{\partial r} \tilde{l}_3 & 0 & 0 \\ -\frac{\mu}{3} \frac{\partial}{\partial r} \tilde{l}_1 & -\frac{\mu}{3} \frac{\partial}{\partial r} \tilde{l}_2 & -\frac{\mu}{3} \frac{\partial}{\partial r} \tilde{l}_3 - \mu\Delta - \tilde{l} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\tilde{l}_5 \end{vmatrix},$$

$$\begin{aligned} \tilde{l}_1 &= l_1 \rho^*, & \tilde{l}_2 &= l_2 \rho^*, & \tilde{l}_3 &= l_3 \rho^*, \\ \tilde{l} &= \rho^* l \rho^*, & \tilde{l}_5 &= \rho^* l_5 \rho^*. \end{aligned}$$

Let $\|\cdot\|$ be the norms in $L^2(\Omega)$, then

$$\|\varphi\|_0 = (\|\tilde{V}_{\lambda}\|^2 + \|\tilde{V}_{\theta}\|^2 + \|\tilde{V}_r\|^2 + \|\tilde{\rho}\|^2 + \|\tilde{T}\|^2)^{1/2}. \quad (22)$$

In $H_0(\Omega)$, we can use the following equivalent norm

$$\|\varphi\|_0 = (\|\tilde{V}_{\lambda}\|^2 + \|\tilde{V}_{\theta}\|^2 + \|\tilde{V}_r\|^2 + \|\rho^*\|^2 + \|\tilde{T}\|^2)^{1/2}. \quad (23)$$

Let

$$|\varphi|_0 = (\|\tilde{V}_{\lambda}\|^2 + \|\tilde{V}_{\theta}\|^2 + \|\tilde{V}_r\|^2 + \|\tilde{T}\|^2)^{1/2}. \quad (24)$$

Let $H_1(\Omega)$ be the complete space with the norm

$$\|\psi\|_1 = (\|V_{\lambda}^*\|_{H^1}^2 + \|V_{\theta}^*\|_{H^1}^2 + \|V_r^*\|_{H^1}^2 + \|\rho^*\|^2 + \|T^*\|_{H^1}^2)^{1/2}, \quad (25)$$

where $\psi = (V_{\lambda}^*, V_{\theta}^*, V_r^*, \rho^*, T^*)'$, $\|\cdot\|_{H^1}$ takes $H^1(\Omega)$ -norm. Here $H^1(\Omega)$ is the standard Sobolev space. And let

$$|\psi|_1 = (\|V_{\lambda}^*\|_{H^1}^2 + \|V_{\theta}^*\|_{H^1}^2 + \|V_r^*\|_{H^1}^2 + \|T^*\|_{H^1}^2)^{1/2}. \quad (26)$$

According to the law of conservation of mass

$$\int_{\Omega} \rho d\Omega = M_{\rho} = \text{const.}, \quad (27)$$

we have

Lemma 1. There exists constant $C > 0$ such that

$$\|\varphi\|_0^2 \leq C\|\psi\|_0^2, \quad (28)$$

$$|\varphi|_0^2 \leq C|\psi|_0^2, \quad (29)$$

$$\forall \varphi = (\tilde{V}_{\lambda}, \tilde{V}_{\theta}, \tilde{V}_r, \tilde{\rho}, \tilde{T})', \psi = (V_{\lambda}^*, V_{\theta}^*, V_r^*, \rho^*, T^*)' \in H_0(\Omega).$$

Lemma 2. There exists constant $C_1 > 0$ such that

$$C_1|\psi|_1^2 \leq (\tilde{L}\psi, \psi), \quad (30)$$

$$\forall \psi = (V_{\lambda}^*, V_{\theta}^*, V_r^*, \rho^*, T^*)' \in H_1(\Omega).$$

The operator $L(\varphi)$ represents dissipative effect in the equation. According to Eq. (16) and Eq. (19), the positive definite self-adjoint property of $L(\varphi)$ shows that due to the effect of dissipation the energy always dissipates. Equations (15) and (20) show that the operator $N(\varphi)$ makes no contribution to the total energy and this is independent of the property of the function φ . The anti-adjoint property of $N(\varphi)$ indicates the important physical essence that the advection effect, the spherical effect of the earth, the Coriolis force and the gravitation etc. do not change the total energy.

The above discussions show that Eq. (11) deduced from the equations of atmospheric motion reflects in concentration the dissipative property of the atmospheric motion with the external forcing. Under the adiabatic and the non-frictional, there is the conservation of energy

$$\frac{d}{dt} \|\varphi\|_0^2 = 0, \quad (31)$$

namely

$$\|\varphi\|_0^2 = \int_{\Omega} \left(\frac{V_{\lambda}^2 + V_{\theta}^2 + V_r^2}{2} + \Phi + C_v T \right) \rho d\Omega = \text{const.} \quad (32)$$

V. EXISTENCE OF ATTRACTOR

Theorem 1. Any solution φ of Eqs. (11) and (12) satisfies

$$\|\varphi(t)\|_0^2 + 2C_1 \int_0^t |\tilde{K}\varphi(t)|_1 dt \leq \|\varphi_0\|_0^2 + 2 \int_0^t (\xi(\varphi, t), \varphi(t)) dt, t \in [0, T], \text{ a. e.} \quad (33)$$

where C_1 is given by Eq. (30), $\tilde{K} = \text{diag}(1/\rho^*, 1/\rho^*, 1/\rho^*, 1/\sqrt{\Phi}, 1/\rho^*)$.

Furthermore, according to Lemmas 1, 2 and the Gronwall inequality, we have

Theorem 2. Any solution φ of Eqs. (11) and (12) satisfies

$$\|\varphi\|_0^2 \leq \{\|\varphi_0\|_0^2 + 2 \int_0^t e^{\tilde{C}t} (C_m + |\zeta(t)|) dt\} e^{-\tilde{C}t}, t \in [0, T], \quad (34)$$

where \tilde{C} and C_m are the positive constants, $|\zeta(t)| = \int_{\Omega} \{|\varepsilon(t)| + |C_K \alpha_s T_s(t)|\} d\Omega$.

Equation (34) implies that the atmosphere system has the characteristic in the decay of the effect of initial field. The long-range evolution of the system is dependent on the variation of the external forcing. In reality, the external forcing should be the bounded, namely, we can have the following assumption

$$0 < |\zeta(t)| \leq M < \infty. \quad (35)$$

Thus we get

$$\|\varphi\|_0^2 \leq \|\varphi_0\|_0^2 e^{-\tilde{C}t} + \tilde{M}(1 - e^{-\tilde{C}t}), \quad (36)$$

where $\tilde{M} = 2(C_m + M)/\tilde{C}$.

Theorem 3. Under the assumption of Eq. (35), the solutions of Eqs. (11) and (12) satisfy, and there exists a bounded absorbing set B_K such that

- (1) if $\varphi_0 \in B_K$, then $\varphi(t) \in B_K$ for $\forall t \geq 0$;
- (2) if $\varphi_0 \notin B_K$, then there exists a $\tau > 0$ such that $\varphi(t) \in B_K$ for $\forall t \geq \tau$.

Proof. Let

$$B_K = \{\varphi = (\tilde{V}_\lambda, \tilde{V}_\theta, \tilde{V}_r, \tilde{\rho}, \tilde{T})' \in H_0(\Omega) \mid \|\varphi\|_0^2 \leq K\}, \quad (37)$$

where $K > \tilde{M}$.

If $\varphi_0 \in B_K$, then

$$\|\varphi_0\|_0^2 \leq K.$$

Using Eq. (36) we have

$$\|\varphi\|_0^2 \leq K e^{-\tilde{C}t} + \tilde{M}(1 - e^{-\tilde{C}t}) \leq K.$$

So we get $\varphi(t) \in B_K$ for $\forall t \geq 0$.

If $\varphi_0 \notin B_K$, then

$$\|\varphi_0\|_0^2 > K.$$

Let

$$\tau = \frac{1}{\tilde{C}} \ln \frac{\|\varphi_0\|_0^2 - \tilde{M}}{K - \tilde{M}}. \quad (38)$$

When $\forall t \geq \tau$, by use of Eq. (36) we obtain

$$\|\varphi_0\|_0^2 \leq \frac{K - \tilde{M}}{\|\varphi_0\|_0^2 - \tilde{M}} \|\varphi_0\|_0^2 + \tilde{M}(1 - \frac{K - \tilde{M}}{\|\varphi_0\|_0^2 - \tilde{M}}) = K.$$

So we have $\varphi(t) \in B_K$. The proof is complete.

Theorem 3 shows that all solutions of (11) and (12) will run into the global absorbing set B_K . Once the solution runs into B_K , it will stay in it forever and not run away from it. The state denoted by points outside of the B_K has only transient sense. The long-range behavior of the system will only depend on the bounded sphere.

Equations (11) and (12) define the continuous mapping $S(t) : H_0 \rightarrow H_0$ such that $S(t)\varphi_0 = \varphi(t)$. We define

$$S(t)R = \{S(t)\varphi_0 \mid \forall \varphi_0 \in R \subset H_0\}. \quad (39)$$

It is easy to see that $S(t)$ is a semigroup.

We define

$$A = \bigcap_{s \geq 0} \overline{\bigcup_{t \geq s} S(t)B_K}. \quad (40)$$

Then we have

Theorem 4. The set A satisfies

- (1) A is a bounded set in H_0 ;
- (2) A is a functional invariant set of the semigroup $S(t)$;
- (3) There exists open neighborhood U of A such that for any $\varphi_0 \in U$ one has $S(t)\varphi_0 \rightarrow A$ as $t \rightarrow \infty$;
- (4) A uniformly attracts the set B_K ;
- (5) A is a global attractor of the semigroup $S(t)$.

The theorem 4 shows that the atmosphere system will trend towards the global attractor A as the time $t \rightarrow \infty$; that is to say, the asymptotic behavior of its solutions

shows itself on the structure of attractor. The attractor A represents the final state of the system. We call it the atmosphere attractor. Therefore, the results deduced from the large-scale atmosphere (Wang et al. 1989; Lions et al. 1992; Li and Chou 1996 a; 1996 b) are extended to the general atmosphere.

VI. SUMMARY

In this paper, based on the full primitive equations of the atmosphere, the equivalent operator equation (11) is derived, which reflects in concentration the dissipative property of the atmospheric motion under the thermal forcing and implies the characteristics of decay of the effect of initial field and nonlinear adjustment of system to the external forcing. Under the basic functional space, the properties and physical senses of the operators are discussed, and it is revealed that the characteristics of long-range motion of system are dependent on the energy dissipation and the energy supplement.

According to the operator equation and the properties of the operators, the existence theorem of the atmosphere attractor under the assumption of the bounded external forcing is given. This shows that all states of the atmosphere system will trend towards the global attractor A as the time increases, and that the global attractor A is just the final set of system as $t \rightarrow \infty$. The characteristics of long-range evolution of the system is implied in the global attractor A . There can be many domains of attraction in the final set and every domain has its own characteristic. It is helpful for catching hold of the long-range evolution of the system to have a clear understanding of the following problems: How can one carry out the effective macroscopic description for the final set of system? How can one get the varied characteristics of the attractor and the distribution situation of attraction domains under the concrete external forcing? etc. These problems await intensive study.

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